HARYANA VISHWAKARMA SKIL UNIVERSITY

(CLASS NOTES)

SUBJECT: EDSC-101(APPLIED MATHEMATICS-I)

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UNIT-1: SETS, RELATIONS AND FUNCTIONS

Set: A Set is a well-define collection of objects or elements.

Examples:

1. The collection of all boys in your class.
2. The collection of all the months of a year beginning with the letter J = {January, June, July}.

But:

3. The collection of ten most talented writers of India is NOT a Set, because it is not "a well defined collection of objects".
4. A team of eleven best-cricket batsman of the world is NOT a Set, because it is not "a well defined collection of objects".

Note:

- Sets are usually denoted by capital letters A, B, S, X, Y etc.
- The elements of a set are represented by small letters *a, b, c, s, x, y, z,* etc.
- If *a* is an element of a set A, then we say that "*a belongs to* A" and write as "$a \in A$".
- If *a* is not an element of a set A, then we say that "*a does not belongs to* A" and write as "$a \notin A$".

Some other examples of sets used in Mathematics:

N : The set of all natural numbers = {1,2,3,…..}

Z : The set of all integers = $\{0, \pm 1, \pm 2, \pm 3, ...\}$

Q : The set of all rational numbers = $\{x : x = \frac{a}{b}, a, b \in Z \text{ and } b \neq 0\}$

R : The set of real numbers = $(-\infty, +\infty)$

Representation of Set: There are two methods of representing a set:

1. Roster or tabular form.

Example: The set of all even positive integers less than 7 is described in roster form as {2, 4, 6}.

2. Set-builder form.

Example: The set of all vowels in the English alphabet is described in set-builder form as
$\{x : x \text{ is a vowel in English alphabet}\}$

Important Examples:

1. Write the set A = {1, 4, 9, 16, 25,….} in set-builder form.

Answer: We may write the set A = {x : x is the square of a natural number}

Or, we can write A = {x : x = n^2, where $n \in$ N}

2. Write the set A = {x : x is a letter of the word MATHEMATICS} in roster form.

Answer: A = {M, A, T, H, E, I, C, S}

Types of Sets

The Empty Set: A set which does not contain any element is called the empty set or the null set.

Symbol: { } or ϕ (phi)

Example:

1. Let A = {Students enrolled in Mechatronics and Manufacturing Courses in your class}. Then A is the empty set.

   Therefore, A = ϕ.

2. Let A = {x: 3 < x < 4, x is a natural number}. Then A is the empty set, because there is no natural number between 3 and 4.

   Therefore A = ϕ.

Finite and Infinite Sets: A set which is empty or consists of a definite number of elements is called finite otherwise, the set is called infinite.

Examples:

1. Let A be the set of the days of the week. Then A is finite.

2. Set of all natural numbers N = {1, 2, 3, 4,….} is Infinite.

Equal Sets: Two sets A and B are said to be equal if they have exactly the same elements and we write A = B.

Otherwise, the sets are said to be unequal and we write A $\neq$ B.

Examples:

Let A = {1, 2, 3, 4} and B = {3, 1, 4, 2}. Then A = B.

Subsets: A set A is said to be a subset of a set B if every element of A is also an element of B.
We write $A \subset B$ if whenever $a \in A$, then $a \in B$.

Therefore, $A \subset B$ if $a \in A \Rightarrow a \in B$

We read the above statement as "A is a subset of B if a is an element of A implies that a is also an element of B".

Examples: Let A = {1, 2, 3} and B = {1, 2, 3, 4, 5, 6}
Then, A is subset of B. we write as $A \subset B$.

Note:

1. $A \subset B$ and $B \subset A \Leftrightarrow B = A$.

2. If A is not a subset of B, we write $A \not\subset B$.

Example: Let A = {a, e, i, o, u} and B = {a, b, c, d, e, f}.
Then A is not a subset of B, also B is not a subset of A.

3. ϕ is subset of every set.

4. Let A and B be two sets. If $A \subset B$ and $A \neq B$, then A is called a proper subset of B and B is called superset of A.

Example: Let A = {1, 2, 3} and B = {1, 2, 3, 4}
then A is proper subset of B and B is superset of A.

5. It is evident that $N \subset Z \subset Q \subset R$.

Closed and open intervals of R:

1. Closed: $[a, b] = \{x: a \leq x \leq b\}$

2. Open: $(a, b) = \{x: a < x < b\}$

Power Set: The collection of all subsets of a set A is called the power set of A. It is denoted by P(A).
In P(A), every element is a set.

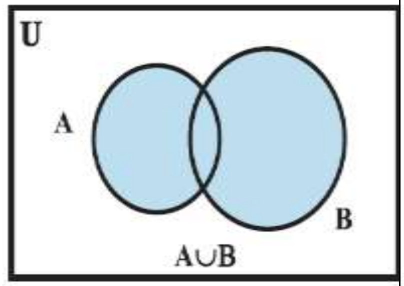
Example: Let A = {1, 2, 3} total 3 elements
Then P(A) = {ϕ, {1,2,3}, {1},{2},{3},{1,2},{1,3},{2,3}}. Total 8 elements.

Note: If number of elements in A = n(A) = m
Then, number of elements in Power set of A = n[P(A)] = 2^m.

Universal Set: A universal set is a set which contains all elements, including itself.

Example: The set of real number R is a universal set.

Operations on Sets



Union of sets: Let A and B be any two sets.
The union of A and B is the set which consists of all the elements of A and all the elements of B, the common elements being taken only once.
The symbol "$\cup$" is used to denote the union. We write A $\cup$ B and usually read as 'A union B'.

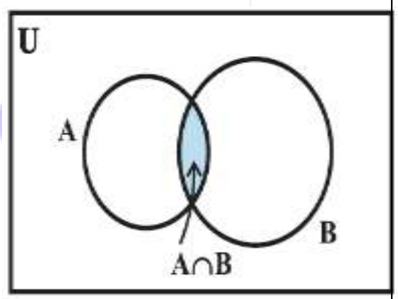
$\Rightarrow$ $A \cup B = \{x : x \in A \text{ or } x \in B\}$.

Examples:

1. Let A = {2, 4, 6, 8} and B = {6, 8, 10, 12}
   Then $A \cup B = \{2, 4, 6, 8, 10, 12\} = B \cup A$

2. Let A = {a, e, i, o, u} and B = {a, i, u}
   Then $A \cup B = \{a, e, i, o, u\} = A$

   This example shows that if $B \subset A$, then $A \cup B = A$.



Intersection of sets: The intersection of sets A and B is the set of all elements which are common to both A and B.
The symbol "$\cap$" is used to denote the intersection.
The intersection of two sets A and B is the set of all those elements which belong to both A and B.

$\Rightarrow$ $A \cap B = \{x : x \in A \text{ and } x \in B\}$.

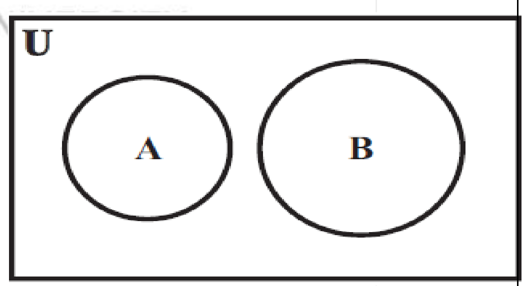
Examples:

1. Let A = {2, 4, 6, 8} and B = {6, 8, 10, 12}.
   Then $A \cap B = \{6, 8\} = B \cap A$

2. A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and B = {2, 3, 5, 7}.
   then $A \cap B = \{2, 3, 5, 7\} = B$. We note that $B \subset A$ and that $A \cap B = B$.

Note: Two sets A and B are said to be mutually disjoint sets if $A \cap B = \phi$

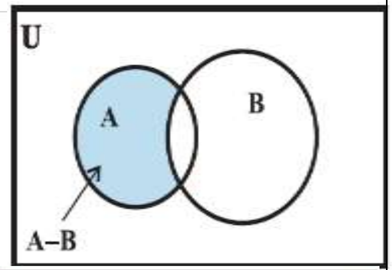
Example: Let A = {a, b, c} and B = {x, y, z} then
$A \cap B = \phi$
Therefore A and B are disjoint sets.



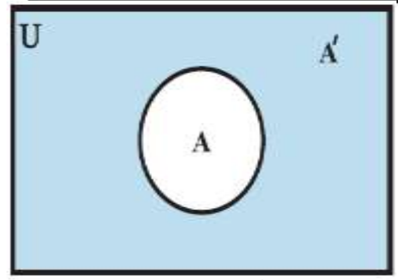
<u>Difference of sets:</u> It is the set of elements which belong to A but not to B.
Symbol: A-B (A minus B)

$\Rightarrow$ A-B $=\{x : x \in$ A and $x \notin$ B$\}$

<u>Example:</u> Let A = {1, 2, 3, 4, 5, 6} and B = {2, 4, 6, 8} then,
A-B = {1, 3, 5}
and B-A = {8}



<u>Complement of a Set:</u> Let U be the universal set and A is a subset of U. Then the complement of A is the set of all elements of U which are not the elements of A. it is denoted by A'



$\Rightarrow$ $A' = \{x : x \in$ U and $x \notin$ A$\}$
Obviously A′ = U-A.

<u>Example:</u> Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and A = {1, 3, 5, 7, 9} then
A′ = {2, 4, 6, 8, 10}
Also (A′)′ = {1, 3, 5, 7, 9} = A

<u>Note:</u>
1. (A′)′ = A
2. U′ = ϕ

<u>Example:</u> Let U = {1, 2, 3, 4, 5, 6}, A = {2, 3} and B = {3, 4, 5}
then A′ = {1, 4, 5, 6} and B′ = {1, 2, 6}
Therefore $A' \cap B'$ = {1, 6}

Also $A \cup B$ = {2, 3, 4, 5}, so that $(A \cup B)'$ = {1, 6}
Therefore $(A \cup B)'$ = {1, 6} = $A' \cap B'$

Similarly $A \cap B$ = {3}, so that $(A \cap B)'$ = {1, 2, 4, 5, 6}

And $A' \cup B'$ = {1, 2, 4, 5, 6}
Therefore $(A \cap B)'$ = {1, 2, 4, 5, 6} = $A' \cup B'$

<u>Cartesian product of Sets:</u>

<u>Cartesian Products of Sets:</u> Given two non-empty sets A and B.
Then Cartesian product $A \times B$ is set of ordered pairs of elements from A and B.

$\Rightarrow$ $A \times B = \{(a, b) : a \in A, b \in B\}$

If either A or B is the null set, then $A \times B$ will also be empty set $\Rightarrow$ $A \times B = \phi$

<u>Example:</u> Let A = {1, 2, 3, 4}, total elements = 4
and B = {a, b, c}, total elements = 3

Then,

$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c), (3, a), (3, b), (3, c), (4, a), (4, b), (4, c)\}$

Note:

1. If number of elements in A = $n(A) = m$ and number of elements in A = $n(B) = n$,
   Number of elements in $A \times B = n(A \times B) = mn$

Introduction to Relations

Relations: A relation R from a non-empty set A to a non-empty set B is a subset of the Cartesian product $A \times B$.

Therefore, $R \subseteq A \times B$.

Note:

1. The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in $A \times B$.

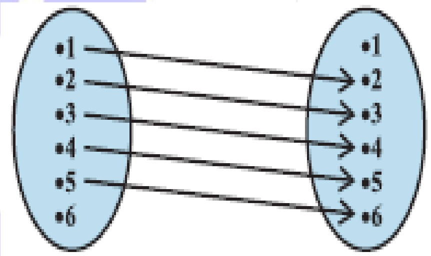
2. The second elements are called the image of the first elements.

Example: Let A = {1, 2, 3, 4, 5, 6}. Define a relation R from A to A by
$R = \{(x, y): y = x + 1\}$
Depict this relation using an arrow diagram.

Solution: By the definition of the relation,

$R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$

The corresponding arrow diagram is described in the figure:



Note: The total number of relations that can be defined from a set A to a set B is the number of possible subsets of $A \times B$.

If $n(A) = m$ and $n(B) = n$, then $n(A \times B) = mn$
and the total number of relations is 2^{mn}.

Example: Let A = {1, 2} then $n(A) = 2$ and B = {3, 4, 5}, then $n(B) = 3$
Therefore,
$A \times B = \{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)\}$
Then $n(A \times B) = 6$
and the number of subsets of $A \times B = 2^6$.
Therefore, the number of Relations from A into B will be 2^6.

Introduction to Functions

Function: A relation from a set X to a set Y is said to be a function (*f*) if every element of set X has one and only one image in set Y.

If f is a function from X to Y and $(x, y) \in f$,
then $f(x) = y$, where y is called the *image* of x under f
and x is called the *preimage* of y under f.

The function (*f*) from X to Y is denoted by:

$$f : X \to Y$$

Example: Examine each of the following relations given below and state in each case, giving reasons whether it is a function or not?

(i) R = {(2, 1), (3, 1), (4, 2)}
(ii) R = {(2, 2), (2, 4), (3, 3), (4, 4)}
(iii) R = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7)}

Answer: (i) Since 2, 3, 4 are the elements of domain of R having their unique images, this relation R is a function.

(ii) Since the same first element 2 corresponds to two different images 2 and 4, this relation is not a function.

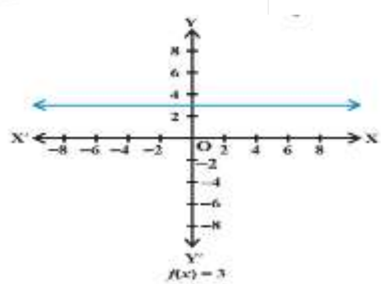
(iii) Since every element has one and only one image, this relation is a function.

Types of Functions – Linear, Quadratic and Polynomial

Polynomial Function: A function $f : R \to R$ is said to be polynomial function if for each x in R, $y = f(x) = a_0 + a_1 x + a_2 x^2 + ... + a_n x^n$ Where, n is a non-negative integer and $a_0, a_1, a_2 ....., a_n \in R$

Example: The functions defined by $f(x) = 8x^2 - 3x + 1$ (Degree = 2) and $g(x) = x^{10} - \sqrt{3}x^4 + x^2 - 13$ (Degree = 10) are some examples of polynomial functions, whereas the function h defined by $h(x) = x^{2/3} + \frac{4}{7}x + 1$ is not polynomial function.

Some functions and their graphs:



Constant function: Define the function $f : R \to R$ by
$y = f(x) = c, x \in R$
Where, c is a constant and each $x \in R$.

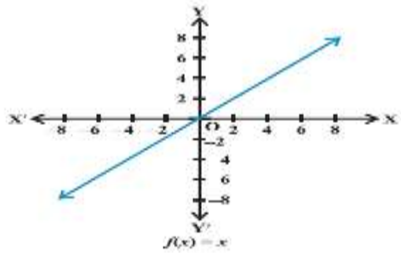
Linear Function: Polynomial functions of degree 1 are called linear function.
Therefore function of the form,

$$y = f(x) = a_0 + a_1 x$$

$$a_0,\ a_1 \in \mathrm{R}$$

For example:

Identity Function: Let $f : \mathrm{R} \to \mathrm{R}$ be a real valued function define as:



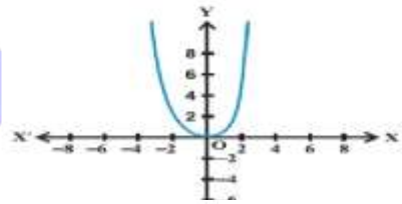
$$y = f(x) = x \text{ for each } x \in \mathrm{R}.$$

Such a function is called the identity function.
Here the domain and range of f are R.
The graph is a straight line which passes through the origin.

Quadratic Function: Polynomial functions of degree 1 are called linear function.

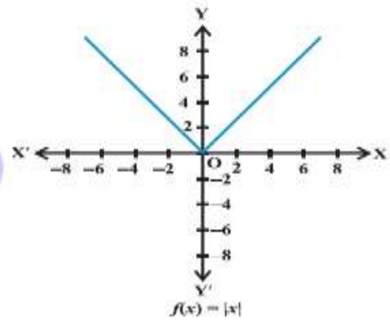
Therefore function of the form, $y = f(x) = a_0 + a_1 x + a_2 x^2,\quad a_0,\ a_1, a_2 \in \mathrm{R}$

For example Let $f : \mathrm{R} \to \mathrm{R}$ be a real valued function define as:



(Parabola): $y = f(x) = x^2$ for each $x \in \mathrm{R}$.

The Modulus Function: The function $f : \mathrm{R} \to \mathrm{R}$ defined by
$f(x) = |x|$ for each $x \in \mathrm{R}$ is called modulus function.



For each non-negative value of x, $f(x)$ is equal to x.
But for negative values of x, the value of $f(x)$ is the negative of the value of x.

Therefore, $f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$